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The effects of environmental decoherence on a mass center position of a macroscopic body are studied using the linear quantum Boltzmann equation. It is shown that under realistic laboratory conditions these effects can be essentially eliminated for dust particles containing 10^{15} atoms. However, the initial maxwellian velocity distribution puts limits on interference type measurements restricting a diameter of a particle to at most 10 nm. The results are illustrated by the analysis of the recent experiments involving C_{60} and C_{70} fullerenes.

Despite its long history the problem of transition between macroscopic and microscopic worlds remains a fundamental issue in the discussion of foundations of quantum mechanics [1,2]. The simplest model ilustrating this topic is a macroscopic body moving in a slowly varying gravitational potential. To describe its motion we use the position of its mass center $\mathbf{x}(t)$ as a collective variable. One expects that for all practical purposes $\mathbf{x}(t)$ is well-localized and evolves according to the Newton equation

$$\frac{d^2}{dt^2}\mathbf{x}(t) = -\nabla U(\mathbf{x}(t)) \tag{1}$$

while apparently quantum delocalized states corresponding to macroscopically extended wave packets $\Psi(\mathbf{x}, t)$ satisfying the Schrödinger equation

$$-i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{x}, t) = -\frac{\hbar^2}{2M}\Delta\Psi(\mathbf{x}, t) + V(\mathbf{x})\Psi(\mathbf{x}, t) \tag{2}$$

do not appear ($V(\mathbf{x}) = MU(\mathbf{x})$).

In the literature there are discussed at least four types of mechanisms leading to localization phenomena (or wave function collapse) of above.

1) *Environmental decoherence.* Quantum coherence is destroyed by scattering processes with particles of an environment both massive and massles (photons)[3]. Emission and absorption of thermal photons must be also included.

2) *Decoherence by Bremsstralung.* Electric charges moving in a slowly varying potential decohere by emission of soft photons [4]

3) *Wave function collapse by gravity.* Here the exact mechanism is not known due to the absence of an ultimate theory of quantum gravity but several models were proposed [5].

4) *Spontaneous localization theories.* Fundamental stochastic or/and nonlinear modifications of the Schrödinger equation are proposed which are neglible at the atomic scale but become relevant for macroscopic bodies [6].

In the following we propose (a more accurate than in [3]) description of the first, most conventional mechanism, and the only one which is very sensitive to the temperature and the density of environmental particles. Denoting by $\mathbf{X}$ and $\mathbf{P}$ the operators of mass center and total momentum of the body we have for any wave vector $\mathbf{k}$

$$e^{i\mathbf{kX}}\mathbf{P}e^{-i\mathbf{kX}} = \mathbf{P} + \hbar\mathbf{k}\ . \tag{3}$$

The effect of a collision with a gas particle and emission, absorption or scattering of a photon is a transfer of momentum $\hbar\mathbf{k}$ which changes the total momentum as described by eq.(3) independently of the detailed microscopic mechanism of energy redistribution. Assuming the statistical independence of different momentum transfer events (called simply *collisions*) we propose the following quantum linear Boltzmann equation (a special case of a quantum Markovian master equation [7]) describing time evolution of the reduced density matrix of the center of mass subsystem

$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H, \rho] + L\rho \tag{4}$$

where $H = \frac{1}{2M}\mathbf{P}^2 + V(\mathbf{X})$,

$$L\rho = \int d^3\mathbf{k}\ n(\mathbf{k})\Big(e^{-i\mathbf{kX}}\rho\ e^{i\mathbf{kX}} - \rho\Big) \tag{5}$$

and $n(\mathbf{k})$ is a density of collisions per unit time leading to the momentum transfer $\hbar\mathbf{k}$. Assuming rotational invariance i.e. $n(\mathbf{k}) = n(k), k = |\mathbf{k}|$ we can introduce the following parametrization

$$4\pi k^2 n(k) = \mathcal{N}\nu(k)\ ,\ \mathcal{N} = 4\pi\int_0^\infty dk\ k^2\ n(k)\ . \tag{6}$$

Here $\nu(k)$ is a probability density of collisions and $\mathcal{N}$ their total number per time unit.

The generator L in the position representation reads

$$(L\rho)(\mathbf{x}|\mathbf{y}) = -\gamma(|\mathbf{x} - \mathbf{y}|)\rho(\mathbf{x}|\mathbf{y}) \tag{7}$$

where

$$\gamma(r) = \mathcal{N}\int_0^\infty dk\ \nu(k)\Big(1 - \frac{\sin kr}{kr}\Big)\ . \tag{8}$$

Introducing the average wave vector $\bar{k}$ defined by

$$\bar{k}^2 = \int_0^\infty dk\ k^2\ \nu(k) \tag{9}$$

we obtain simple formulas for the decay rates of the off-diagonal matrix elements $\rho(\mathbf{x}|\mathbf{y})$ in two regimes:

for $|\mathbf{x}-\mathbf{y}| >> \bar{\lambda} = 2\pi\bar{k}^{-1}$

$$\gamma(|\mathbf{x}-\mathbf{y}|) \simeq \mathcal{N} \tag{10}$$

for $|\mathbf{x}-\mathbf{y}| << \bar{\lambda}$

$$\gamma(|\mathbf{x}-\mathbf{y}|) \simeq \mathcal{N}\bar{k}^2\ |\mathbf{x}-\mathbf{y}|^2\ . \tag{11}$$

In order to analyse an experiment which takes time t between the preparation of a quantum state and its measurement it is convenient to introduce two new parameters:

$$\text{coherence factor} \quad \Gamma = e^{-t\mathcal{N}} \tag{12}$$

and

$$\text{coherence length} \quad l_{coh} = \frac{\bar{\lambda}}{2\pi\sqrt{t\mathcal{N}}} \tag{13}$$

which, according to eq.(11) gives the maximal distance $|\mathbf{x}-\mathbf{y}|$ such that the corresponding off-diagonal elements decohere less than by a factor e^{-1}. The parameter l_{coh} puts an upper bound on the dimensions of diffraction grating (slit width and period) which can produce interference patterns.

To estimate the effect of thermal photons absorption (or emission) for a macroscopic body of a radius R treated as a black body we calculate $\mathcal{N}$ as a number of photons with Planck density entering a surface of a ball per unit time

$$\mathcal{N} = \frac{1}{4}(4\pi R^2 c)\frac{1}{\pi^2}\int_0^\infty \frac{k^2\ dk}{e^{c\hbar k/k_B T}-1} \simeq 0.8\ R^2 c\Big(\frac{k_B T}{\hbar c}\Big)^3 . \tag{14}$$

The corresponding decoherence time $\tau = \mathcal{N}^{-1}$ is given by

$$\tau[sec] \approx 10^{-17}(R[m])^{-2}(T[K])^{-3}\ . \tag{15}$$

It follows from eq.(15) that the $3K$ background radiation alone washes out all quantum coherence effects described by eq.(2) for the macroscopic bodies with $R >> \bar{\lambda} = 2\pi\hbar c/k_B T \approx 10^{-3}m$.

Consider now a laboratory experiment performed at temperatures of the order of $T \approx 1K$, high vacuum of $n_0 \approx 10^9 particles/m^3$ (mass of the gas particle $m \approx 10^{-25}kg$) and with a "small macroscopic" body , say a metallic ball of a radius $a = 10^{-5}m$ containing $\approx 10^{15}$ atoms. Because at low temperatures the metallic body is almost a perfect conductor and its radius is much smaller than the thermal radiation wavelength the leading decoherence factors are the scattering of a low density gas particles and the Rayleigh scattering of thermal photons. A number of collisions per unit time for the former is given in terms of the average thermal velocity $v_{th} = \sqrt{8k_B T/\pi m}$

$$\mathcal{N}_{gas} = \frac{1}{2}(4\pi a^2\ v_{th})n_0 = 4\sqrt{2\pi}\ a^2\ n_0\ \sqrt{k_B T/m}\ . \tag{16}$$

The Rayleigh scattering is characterized by the k-dependent cross-section [8]

$$\sigma(k) = \frac{10\pi}{3}k^4 a^6\ . \tag{17}$$

and leads to

$$\mathcal{N}_R = \frac{1}{2}\Big(\frac{10\pi}{3}a^6 c\Big)\frac{1}{\pi^2}\int_0^\infty \frac{k^6\ dk}{e^{c\hbar k/k_B T}-1}$$

$$\simeq 380\ c\ a^6\Big(\frac{k_B T}{\hbar c}\Big)^7 \tag{18}$$

A straightforward calculation with the parameters of above yields

$$\mathcal{N}_{gas} \approx 20[sec^{-1}]\ , \mathcal{N}_R \approx 500[sec^{-1}]\ . \tag{19}$$

Both contributions are comparable in this regime and display quite different temperature and radius dependence. Hence in principle the onset of environmental decoherence might be observed and well separated from the other hypothetical mechanisms like *gravitational collapse* and *spontaneous localization* which incidentally are supposed to be of the comparable magnitude for a body containing 10^{15} atoms [5][6](the mass center motion of an electrically neutral body should not produce *Bremsstralung*).

Unfortunately, the main obstacle is now a possibility of preparing and detecting quantum delocalised states. Any interference type experiment demands that the de Broglie wavelength $\Lambda = 2\pi\hbar/MV$ is comparable with the width of the slits d. In all existing experiments starting from the historical Young one till the recent ones performed by Zeilinger group [9] the ratio $\delta = \Lambda/d$ is between $10^{-4} - 1$. Obviously, for diffraction gratings we have a geometrical condition

$$\Lambda = \delta d \geq 2\delta a\ . \tag{20}$$

On the other hand V cannot be smaller than the thermal velocity $V_{th} = \sqrt{8k_B T/\pi M}$ what gives

$$\Lambda \leq \frac{2\pi\hbar}{MV_{th}} = \frac{(\pi)^{3/2}\hbar}{\sqrt{(2Mk_B T)}}\ . \tag{21}$$

Puting $M = (4/3)\pi a^3\kappa$ where the density of the body $\kappa \approx 10^4 kg/m^3$ and a rather optimistic value for $\delta =$

10^{-5} we obtain from eqs.(20,21) the final condition for the successful interference type experiment

$$a \leq \delta^{-2/5}\left(\hbar^2/\kappa k_B T\right)^{1/5} \approx 10(T[K])^{-1/5}[nm] \ . \quad (22)$$

The very weak temperature dependence of the right hand side of eq.(22) makes rather impossible to go essentially far beyond the nm scale with traditional interference measurements.

The recent successful experiments involving C_{60}, C_{70} [9], the molecules with $a \simeq 0.5nm$, lie not far from the border between classical and quantum worlds established by eq.(22).The authors rightfully argued that decoherence effects can be neglible under the conditions of their experiments. We can quite precisely estimate the decoherence magnitude using their data. First we have to compute the total number of collisions $t\mathcal{N}$ during the time of flight t of a fullerene due to emission, absorption and Rayleigh scattering of radiation and scattering of gas particles. For the emission the authors estimate $t\mathcal{N}_1 \sim 3.5$. As the environment temperature $T_1 \approx 300K$ is much lower than the temperature of the fullerene molecule $T_2 \approx 900K$ then due to eq.(14) absorption can be neglected. The same holds due to eq.(18) for the Rayleigh scattering because the radius a is much smaller than the average radiation wavelength $\lambda_{T_1} \approx 10\mu m$. The number of collisions with gas particles is estimated to be $t\mathcal{N}_2 \approx 10^{-2}$ and can be neglected also. As $\bar{\lambda} \approx 10\mu m$ the coherence length (13) $l_{coh} \approx 1\mu m$ – the value which is still essentially larger than the width of the slits (50nm) and their separation (100nm). It follows that the diffraction picture is not destroyed by decoherence which reduces the effective collimation width only.

It was shown that under realistic laboratory conditions the environmental decoherence of the center of mass position can be eliminated on the time scale of *miliseconds* for macroscopic dust particles containing 10^{15} atoms. Nevertheless, the emerging quantum coherence effects are completely overshadowed by the initial maxwellian distribution of the dust particle velocity at least in all standard diffraction - interference type experiments. Therefore, only completely new ideas concerning preparation and measurement of spatially extended quantum states might push the border between quantum and classical worlds beyond the scale of *nanometers*. On the other hand the experiments involving large molecules of a diameter less than $10nm$ are feasible an can provide interesting information concerning the detailed mechanism of environmental decoherence.

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